

### **Wine glass acoustics**

How does the frequency vary with the water amount in a singing wineglass?

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## **Abstract**

This Extended essay explores the following questions: How does the frequency vary with the water amount in a singing wineglass? This paper will focus on how and why the frequency changes as it does.

This investigation is approached through recording the frequencies for each height interval. The frequency for each height interval will be recorded three times, and an average was found. The PC software tool FFT – Properties was used to calculate the frequency of the measurements, and by using Microsoft Excel, we found the regression line of our data's.

Using A.P. French's paper on wineglass acoustics, we were able to prove mathematically and through graphing tools how and why the frequency changed as it does. We also used the program DataStudio to find the standard deviation of the data points compared to the line we should have gotten when we plotted French's equation.

By using French's equation, we could explain that it was the shape of the wineglass that determined the frequency change. The radius of the bowl at the water level and the thickness of the glass wall contributed to a gradual decrease of the frequency at the bottom part, while a steep decrease at the top.

## Table of Contents

Abstract: .....	1
Introduction .....	4
The theory:.....	5
Superposition: .....	5
Standing waves, nodes and antinodes .....	6
Slip and Stick effect: .....	7
How does the glass vibrate?.....	7
Resonance frequency:.....	8
Beating.....	9
Linear regression, Standard deviation and Correlation .....	10
Relationship between the three operations.....	11
A.P. French's general formula .....	12
The experiment .....	13
Variables:.....	13
Procedure and Setup.....	14
Equipment:.....	14
Setup.....	14
Data processing .....	16
Raw data .....	16
Observations .....	16
Processed data.....	17
Data with mass as independent variable .....	17
Processed data according to French's general equation .....	19
Analysis .....	22
Conclusion.....	23
Further investigation .....	23
Evaluation:.....	23
Appendix .....	25
Tables of raw data: .....	25
Bibliography .....	<b>Feil! Bokmerke er ikke definert.</b>

## Introduction

The singing wine glass, also known as glass harp or the Harmonica, is a glass instrument invented in 1714 by Richard Pockrich, and a modern version of the Harmonica is said to be invented by Benjamin Franklin. The sound of this beautiful instrument is created when energy is applied to the glass and the physical properties of the glass start to resonate. The applied energy will then in this case be rubbing one or two wet fingers on the rim of a glass, and the glass will then start to vibrate, at its natural frequency. However, when energy is applied externally, this frequency will be called the resonate frequency. Different tones can be obtained by varying the amount of water inside the glass. Wine glasses are usually used, so that one can hold onto the stem while rubbing to minimize the interference of the singing. So my research question will be:

### How does the frequency vary with the water amount in a singing wineglass?

The approach I will be using to answer my research question will be to measure the frequencies with different amount of water. Then I am going to find a relationship between the frequency outcome and how we vary the water. A.P. French has written a journal paper<sup>1</sup> about this specific type of resonance use on wineglasses, he derived a general formula for how the frequency could vary with the water amount in the glass, so this paper will be used to confirm our experiment. The program *FFT-properties*<sup>2</sup> is the program used to obtain the records taken during the experiment as frequency (Hz) values. By analyzing the different frequencies and process them, we are able to find a relationship between the frequency variation and amount of water used.

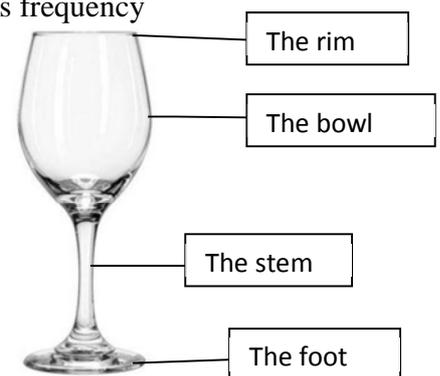


Figure 1 - A wine glass<sup>3</sup>

<sup>1</sup> "In Vino Veritas – A study of wineglass acoustics" A.P French, Am. J. Phys., Vol.51, No. 8, August 1983

<sup>2</sup> <http://www.dewresearch.com/fftp-main.html>

<sup>3</sup> <http://www.webstaurantstore.com/libbey-3057-perception-11-oz-wine-glass-24-cs/libbey-3057-perception-11-oz-wine-glass-24-cs.jpg>

## The Theory:

### Superposition:<sup>4</sup>

When there is more than one wave travelling in the same medium, the sum of the wave functions is the algebraic sum of the values of the wave function. This is called superposition, and is the basic theory behind standing waves. So when waves travel towards each other, instead of breaking the rhythm totally, they travel through each other. There are two types of superposition, constructive, where the net sum of the waves gives a positive displacement, and destructive, vice versa.

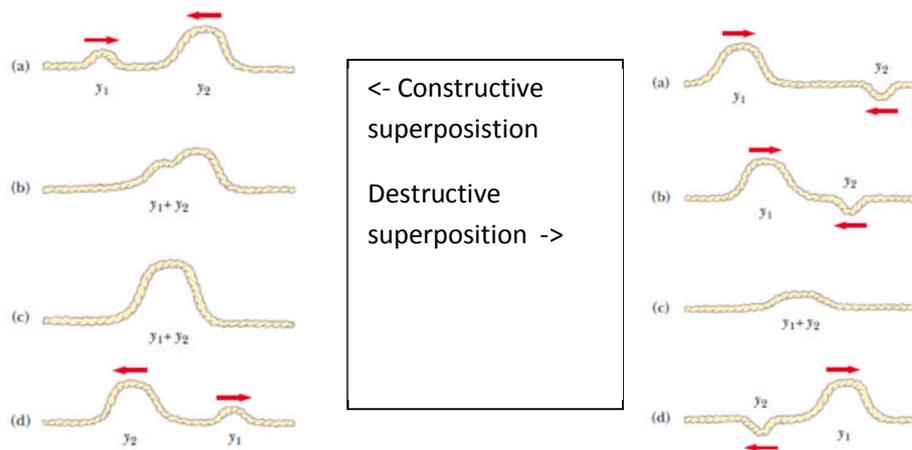


Figure 2 – Superposition

So the net displacement of the wave will be as the picture states:

$$y_{net} = y_1 + y_2$$

[T.1]

Superposition is the basis for the understanding of standing waves, which will be relevant to the observations.

<sup>4</sup> Basic theory from “Physics for Scientists and Engineers 6E by Serway and Jewett” unless otherwise stated

### Standing waves, nodes and antinodes

We can analyze situations where two or more waves travel together as standing waves. Then we can consider two waves that are travelling towards each other, one is then travelling to the right ( $y_1$ ) and one is travelling to the left ( $y_2$ ):

$$y_1 = A \sin(kx - \omega t) \qquad y_2 = A \sin(kx + \omega t)$$

Then applying the principles of superposition and [T.1], then you get:

$$y_{net} = y_1 + y_2 = A \sin(kx + \omega t) + A \sin(kx - \omega t)$$

By using the trigonometric identity [ $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$ ], we get:

$$y_{net} = 2A \sin kx \times \cos \omega t$$

This is the standing wave function. When  $\sin kx = 0$ , then the amplitude of a standing wave motion is at a minimum. These are called zero points, more known as nodes! This is obtained when:

$$kx = \pi, 2\pi, 3\pi \dots$$

Substituting  $k$  with  $k = \frac{2\pi}{\lambda}$ , we get:

$$x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \dots = \frac{n\lambda}{2} \quad n = 1, 2, 3 \dots$$

These zero points can be easily observed during the experiment.

### Slip and Stick effect<sup>5</sup>:

The slip and Stick effect is a frictional effect where two surfaces are dragged along each other to stick together and slips with very short intervals of time. The main theory behind it is that friction between two surfaces wants to hold them together, thus stick. But the external force on acting from one surface on the other is continuously trying to pull the two surfaces away, therefore slip. Then the kinetic friction coefficient becomes larger than the static friction coefficient, and vice versa when the two surfaces are “sticked” together. That is why it is called the stick-and-slip effect. This motion is not a simple harmonic motion. Violin uses this technique to produce the sound, or when you drag your fingers over a balloon, but mainly our singing wine glass uses this to create the sounds. By using this technique, the frictional jerking will create vibrations within the glass wall.

### How does the glass vibrate?<sup>6</sup>

When the glass is affected by the stick-and-slip motion, it starts to move in a very special way. Since glass is quite elastic, the rim starts to deform into an elliptical shape. This is proved by A. P. French<sup>7</sup> through his derived equation of the horizontal radial displacement  $x$ :

$$x(z, \theta, t) = \Delta_0 f(z) \cos 2\theta \cos \omega t$$

Where the  $[\cos 2\theta]$  factor describes the elliptical movement from the circular mean shape. In this case when  $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \dots$ , there will be diagonal nodal lines which are indicated in red lines in Figure. 4.

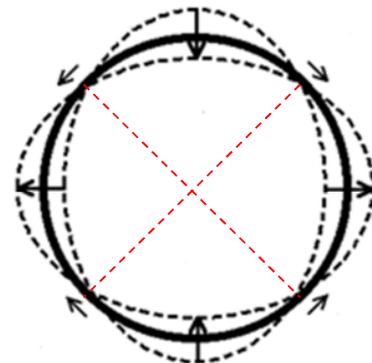


Figure 3 - How the glass vibrates

<sup>5</sup> <http://www.ccmr.cornell.edu/education/ask/index.html?qid=1143>

<sup>6</sup> <http://www.ccmr.cornell.edu/education/ask/index.html?qid=1143>

<sup>7</sup> "In Vino Veritas" page 689.



Figure 4 - A vibrating glass with water in action<sup>8</sup>

We can clearly see the antinodes on this picture. It is where the water has maximum and minimum displacement, crest and trough, these are the antinodes. However, the nodes and antinodes are not fixed in one position, but moving along your finger.

### Resonance frequency:

*“If a periodic force is applied to such a system, the amplitude of the resulting motion is greatest when the frequency of the applied force is equal to one of the natural frequencies of the system.”*<sup>9</sup> This is often referred as resonance, and the resonance frequencies are the frequencies where the system exhibits relatively large amplitudes.

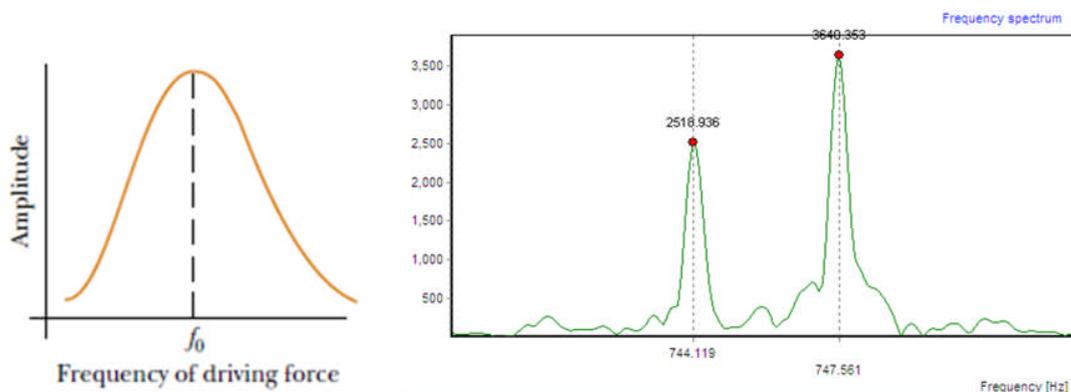


Figure 5 - Amplitude - frequency graph<sup>10</sup>

Figure 6 - Resonance frequencies from 50mm, 3.measurement<sup>11</sup>

<sup>8</sup> <http://www.flickr.com/photos/euicho/273665203/in/photostream/>

<sup>9</sup> Serway, page 558

<sup>10</sup> Images from Serway, p. 558, screen printed

<sup>11</sup> Screen printed image from the FFT-program.

The resonance frequency is essential in this experiment because the sound the wine glass makes when its rim is rubbed by a moistened finger is the resonance frequency sound of the glass with the specific amount of water.

The resonance frequency is also the main theory behind how to break a glass with sound. Shortly summarized, when a sound played off close to the glass matches the resonance frequency and is loud enough, the glass will continuously vibrate until the elastic properties of the glass can no longer handle the energy from the sound flowing into the glass, and hence break.<sup>12</sup>

### Beating

Beating is a result of the superposition of two waves that are slightly out of phase, this will give a periodic variation in the amplitude. Adding the superposition of two wave functions with different frequency, where  $x=0$  will become:

$$y_1 = A \cos \omega_1 t = A \cos 2\pi f_1 t$$

$$y_2 = A \cos \omega_2 t = A \cos 2\pi f_2 t$$

$$y_1 + y_2 = A(\cos 2\pi f_1 t + \cos 2\pi f_2 t)$$

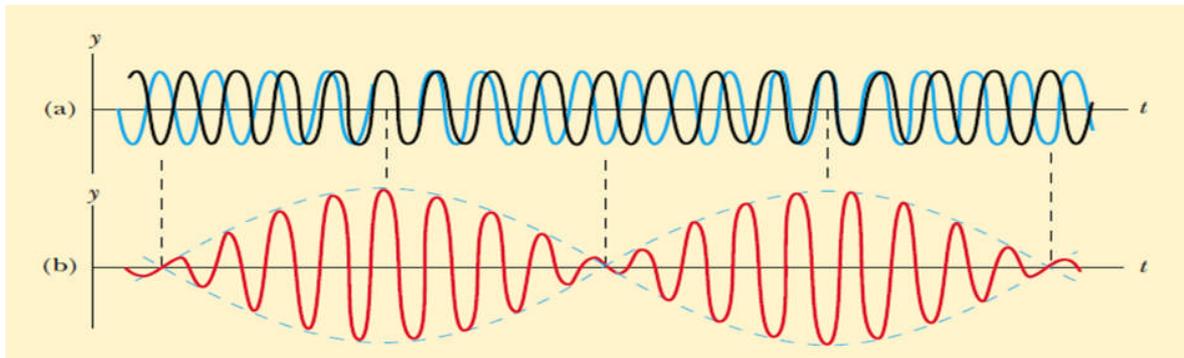


Figure 7 - Beating<sup>13</sup>

Fig. 9 (a): two waves out of phase, one blue ( $y_1$ ) and one black ( $y_2$ )

(b): the superposition of the two waves out of phase ( $y_1 + y_2$ )

<sup>12</sup> <http://www.fas.harvard.edu/~scdiroff/lids/OscillationsWaves/ShatteringWineglass/ShatteringWineglass.html>

<sup>13</sup> Figure of beating from Serway, p.565

Using the trigonometric identity:  $\cos a + \cos b = 2 \cos\left(\frac{a-b}{2}\right) \cos\left(\frac{a+b}{2}\right)$

and rearranging it, we get:

$$A_{resultant} = 2A \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t$$

Through the figure and the equation, one will hear a sound of periodically varying intensity, amplitude due to the cosine factor in the equation.

The beat frequency will then be:

$$f_{beat} = |f_1 - f_2|$$

For example, the frequency outcome of the wineglass empty was 796.3Hz and 800.2Hz. The average sound wave frequency will be 798.25 Hz, which will be the resultant sound wave frequency. There will be a beat frequency of 3.9Hz, so the sound of 798.25Hz will then be heard with an intensity maximum approximately four times per second.

The reason we get beating in our experiment is due to the microphone of the recorder is placed beside the wineglass. When the wineglass vibrates due to the periodically sliding finger on the rim, it will periodically move slightly away and towards the microphone as seen in *Figure 4*. This will cause a periodical variation of the intensity of the sound recorded, and thus the beating theory can be applied to our results. This is also the theory behind why we will get two peaks when we run our data in the FFT. So when processing the data, we can average the two peaks and get the overall frequency.

### **Linear regression<sup>14</sup>, Standard deviation<sup>15</sup> and Correlation<sup>16</sup>**

Linear regression is the operation where it finds the line of best fit for the data points considered. This is done by minimizing the sum of the square of the vertical distance from the result values to the trend line. This operation is used on Excel and DataStudio to find our trend line.

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<sup>14</sup> [http://en.wikipedia.org/wiki/Linear\\_regression](http://en.wikipedia.org/wiki/Linear_regression)

<sup>15</sup> [http://en.wikipedia.org/wiki/Standard\\_deviation](http://en.wikipedia.org/wiki/Standard_deviation)

<sup>16</sup> [http://en.wikipedia.org/wiki/Correlation\\_and\\_dependence](http://en.wikipedia.org/wiki/Correlation_and_dependence)

Standard deviation is a tool used in scientific measurements, statistics and probability to find out how much the results are scattered about the trend line of the values. If the results are close to the trend line, then the standard deviation will be low. If it is a large value, then the data points are more spread out. The standard deviation can be calculated through:

$$\sigma = \sqrt{E[(x - \mu)^2]} = \sqrt{\text{Variance}}$$

$\sigma$  stands for the standard deviation,  $x$  is the x value,  $\mu$  is the mean value, while  $E$  is the operation “the average of the expected values”.

The correlation coefficient shows us the relationship between the two variables considered. Pearson’s correlation coefficient deals with strength of the linear relationship between the two variables considered, but cannot show every characteristic the data points have.  $r$  can be found:

$$r = \frac{E[(x - \mu_x)(y - \mu_y)]}{\sigma_x \sigma_y}$$

$(x - \mu_x)$  represents the horizontal distance from the data point to the regression line, while  $(y - \mu_y)$  is the vertical distance.

### Relationship between the three operations

The relationship between the regression line and the standard deviation is that the slope constant  $a$  in  $y = ax + b$  is directly related to the standard deviation and the correlation coefficient  $r$ , which will be explained soon:

$$a = r \times \frac{\sigma_y}{\sigma_x}$$

This is what the programs automatically give us later in the analysis part.

### A.P. French's general formula<sup>17</sup>

In A.P. French's paper, he presented an equation for variation of frequency variation:

$$\left(\frac{f_0}{f_d}\right)^2 \approx 1 + \frac{\beta\rho_l R}{5\rho_g a} \left(1 - \frac{d}{H^*}\right)^4$$

His general formula was intended to describe an ideal cylindrical glass, but in his paper, he found out that this formula works approximately for any types of glass.

As we can see, the part  $\frac{\beta\rho_l R}{5\rho_g a}$  is build up by different constants

representing different properties of his ideal cylindrical glass,

we can substitute this with  $k$ . The  $\left(1 - \frac{d}{H^*}\right)$  part is representing the variable of this

equation, the height of the water  $d$ . So we can substitute the

$$\left(\frac{f_0}{f_d}\right)^2 = 1 + kX^4$$

So if the wineglass is ideal and cylindrical, the best fit line for plotting  $\left(\frac{f_0}{f_d}\right)^2$  against

$\left(1 - \frac{d}{H^*}\right)^4$  should give us a totally straight and linear line.

$f_0$	= frequency of empty glass
$f_d$	= frequency of partially filled glass (Hz)
$\beta$	= a constant
$\rho_l$	= density of liquid
$\rho_g$	= density of glass
$R$	= radius of water
$a$	= glass thickness
$d$	= distance from top of glass top water
$H^*$	= effective height of glass

Table 1 - Explanation of the signs used in the equation by A.P. French

<sup>17</sup> In *Vino Veritas*, page 692

## The experiment

### Variables:

The glass from IKEA was the only glass used during the whole experiment, so the shape of the glass will not change, and so would not the frequency of the empty glass change, because there would not be any interference to the frequency outcome if there is not water in the glass to disturb the vibrations. Due to the fact that water's volume change with temperature, the water used to the experiments was put away for a while before using it to get it to an approximately room temperature. The frequencies were found by processing the records on *FFT-properties*. There is an input control on the sound recorder, which can be adjusted so that sound within a certain radius can be recorded. Before the actual record, the recorder will show how the device reacts to the different sounds that it catches, so I turned down the input and waited for everything to be completely quite (for the recorder) before I started recording.

Two different experiments are to be done; one where the mass of the water in the wine glass is varied and the other one is the height measured from the bottom upwards. To minimize the uncertainties, a weight with 0.1g accuracy was used for the weight and a piece of spaghetti where every 10cm was marked was used as a ruler that could measure the height of the water from the bottom of the glass. The reason of using spaghetti was to minimize the interference of the water level with the measuring instrument.



Figure 8 - Measuring instrument, spaghetti

**Independent variables:** the height of water in (mm)

**Dependent variables:** the frequency (Hz)

**Controlled variables:** the frequency of an empty wine glass, the shape of wineglass and the water temperature (C°)

## Procedure and Setup

### Equipment:

- Wineglass (IKEA product nr: 000.151.34)
- Water
- Pipette
- Spaghetti straw
- Sound recorder, EDIROL R09
- Processing program, FFT Properties version 5.0



Figure 9 - The equipments

### Setup

We are first going to measure the mass/volume and the frequency outcome, then the heights with the frequency outcome. To measure the exact volume, we use a weight to find the volume, because 1 gram of water = 1 ml of water, the weight has an accuracy of 0.01g. We used a pipette to slowly add drops of water to get the amount of water as accurate as possible.

The intervals chosen to test out the frequency variation is 10g for each interval, the sound played was recorded as a WAV file on an EDIROL R09 sound recorder. Then the FFT program<sup>18</sup> is used to analyse the sound waves. The analysis will come out in an Intensity (amplitude)–Frequency graph, where the two peaks can be found. Using the “peak finder” in the program to find the intensity of the peak and the frequency the peak lies on.

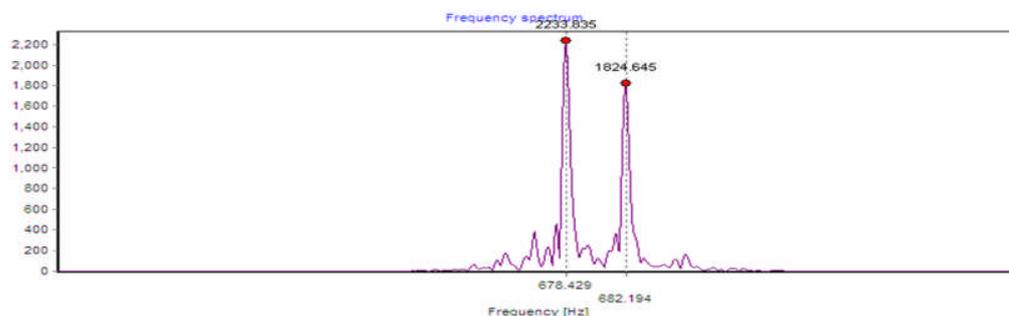


Figure 10 - the two peaks of 60 mm height, 3. measurement

<sup>18</sup> FFT properties version 5.0, trial

The almost same procedure is done with the measurements based on heights; just that the way to measure the heights is through a spaghetti straw. We needed the height of the glass from the exact bottom, so measuring with a ruler outside the glass would give extremely vague accuracy. So I chose to mark the spaghetti with intervals of 10mm each. Then put the spaghetti in the glass and try to hold it as straight as possible, and fill the glass until the marks were reached. Measurements were taken when the water surface seen from below touched the black line that indicated the intervals, the meniscus of the water on the spaghetti straw.

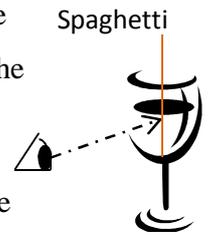


Figure 11 - how the measurements were taken

To be accurate, each interval of investigation is measured three times. Measure all the intervals in the glass once, then start over from the bottom again, and do this three times for both the mass and the heights. The first measurement is done at the student council office at my school, because that room has very good sound proofed, so the sound recording will have minimum disturbance, and this is done when everybody else at the school was having classes, nobody was in the vestibule.

In the end the frequency of an empty glass is measured, but only once is necessary, because it will always be the same, compared to when there is water.

Each measurement lasted around 12-15 seconds, because when analyzing with the FFT, we analyze the sound over a period of time, which I chose to be 3 seconds, so that any random interference can be avoided by analyzing the sound from different parts of the record.

The glass was dried properly with paper before starting on another measurement, to avoid as much uncertainty as possible.

## Data processing

### Raw data<sup>19</sup>

Table 2 - Raw data with height as independent variable:

Height (mm) $\pm 0.5$ mm	peak1 (Hz) $\pm 0.05$ Hz	peak2 (Hz) $\pm 0.05$ Hz
1. measurement		
0	796.3	800.2
10	796.1	798.9
20	795.3	798.1
30	791.4	794.3
40	778.0	781.2
50	742.9	745.9
60	676.8	679.7
70	582.0	585.0
80	477.5	480.8
90	375.4	378.5

The frequencies were found by analysing the sound waves over a period of time, which I chose to be approximately four seconds. The frequency was displayed with three decimals; I rounded it up to one decimal. The effective height of the glass is 90mm, total height

### Observations

The further up the water was, the easier the vibration waves were to be seen. You could see the waves close to the glass walls, and standing wave patterns can be observed. They occur 90degree to each other like: The circle is the glass, it is the cross (+) that copies my moves with my fingers when playing. These wave patterns are the nodes of this motion described in the theory section.

The sound varied in intensity periodically, so it sounded like it has a certain beat, rhythm of playing off one pitch.

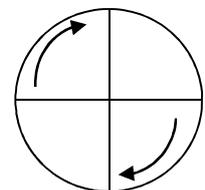


Figure 12 - How the wave moves

<sup>19</sup> In total, three measurements were taken for each independent variable, they can be found in the appendix

## Processed data

### Data with mass as independent variable

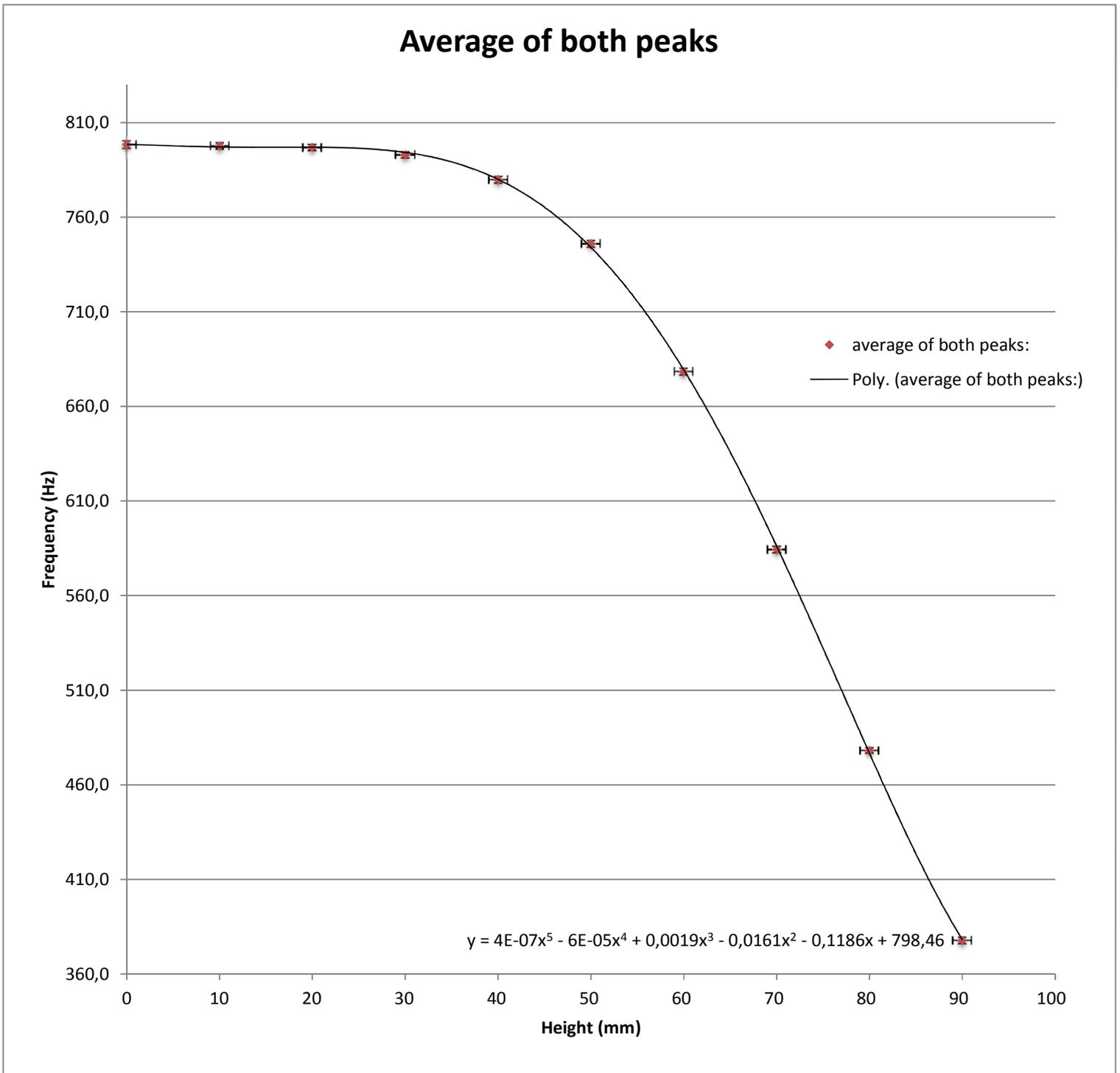
#### *Average of the data's*

Height (cm) $\pm$ 0.5 cm	Average of peak 1 (Hz)	Peak 1 Unc. (Hz)	Average peak 2 (Hz)	Peak 2 Unc. (Hz)	Average of both peaks (Hz)	Av. Of both peaks Unc. (Hz)
0	796.3	0.1	800.2	0.1	798.3	2.0
10	796.0	0.1	799.2	0.3	797.6	1.6
20	795.4	0.5	798.4	0.3	796.9	1.5
30	791.5	0.2	794.7	0.4	793.1	1.6
40	778.2	0.6	781.6	0.5	779.9	1.7
50	744.4	1.6	747.8	1.9	746.1	1.7
60	676.8	1.6	680.2	1.7	678.5	1.7
70	582.6	2.4	586.1	2.2	584.4	1.8
80	476.7	3.3	479.7	2.9	478.2	1.5
90	376.1	0.8	379.4	0.9	377.7	1.7

**Table 3 - Processed data with height as independent variable**

The uncertainty of the height is estimated  $\frac{1}{2}$  of the measuring instrument.

*Graph of average of both peaks*



Graph 1 - Average of both peaks, height as independent variable

**Processed data according to French's general equation**

According to A.P. French, if we plot  $\left(\frac{f_0}{f}\right)^2$  against  $\left(1 - \frac{d}{H^*}\right)$  in a graph, we should get a straight line, so we first process the data:

$h$ (mm) $\pm$ 0.5mm	$f$ (Hz)	$\Delta f$ (Hz)	$\left(\frac{f_0}{f}\right)$	$\Delta\left(\frac{f_0}{f}\right)$	$\left(\frac{f_0}{f}\right)^2$	$\Delta\left(\frac{f_0}{f}\right)^2$	$\left(1 - \frac{d}{H^*}\right)^4$	$\Delta\left(1 - \frac{d}{H^*}\right)^4$
0	789.3	2.0	1.000	0.005	1.000	0.01	0.0000	Error
10	797.6	1.6	0.990	0.004	0.979	0.01	0.0001	6E-05
20	796.9	1.5	0.990	0.004	0.981	0.01	0.0023	4E-04
30	793.1	1.6	0.995	0.004	0.990	0.01	0.0118	1E-03
40	779.9	1.7	1.012	0.005	1.024	0.01	0.0373	3E-03
50	746.1	1.7	1.058	0.005	1.119	0.01	0.0911	5E-03
60	678.5	1.7	1.163	0.006	1.353	0.01	0.1890	8E-03
70	584.4	1.8	1.351	0.007	1.824	0.02	0.3501	1E-02
80	478.2	1.5	1.651	0.009	2.724	0.03	0.5973	2E-02
90	377.7	1.7	2.090	0.014	4.367	0.06	0.9568	2E-02

Table 4 - Processed data 2

$h$  is the height of the water from bottom of,  $f$  is the frequency at the different heights,  $f_0$  is the frequency of the glass when it is empty,  $d$  is the vertical distance from the top of the glass to the water surface and is found by  $(H^* - h)$ ,  $H^*$  is the effective height of the glass which is 91 mm, and  $\Delta$  represents uncertainty of.

Through the STDEVA[value1,value2...] operation in Excel, it found us the standard deviation for the x and y variables.  $\sigma_y = 1.1096294$  and  $\sigma_x = 0.3232$ .

Uncertainty of  $\left(\frac{f_0}{f}\right)$  is found through:

$$\Delta Q = Q \left( \frac{\Delta f_0}{f_0} + \frac{\Delta f}{f} \right)$$

where  $\Delta Q$  is the final uncertainty,  $\Delta f_0$  is the uncertainty of the frequency with an empty glass and  $\Delta f$  is the uncertainty of frequency with different amount of water.

Uncertainty of  $\left(\frac{f_0}{f}\right)^2$  is found through:

$$\Delta Q = Q \left( 2 \times \frac{\Delta \left( \frac{f_0}{f} \right)}{\left( \frac{f_0}{f} \right)} \right)$$

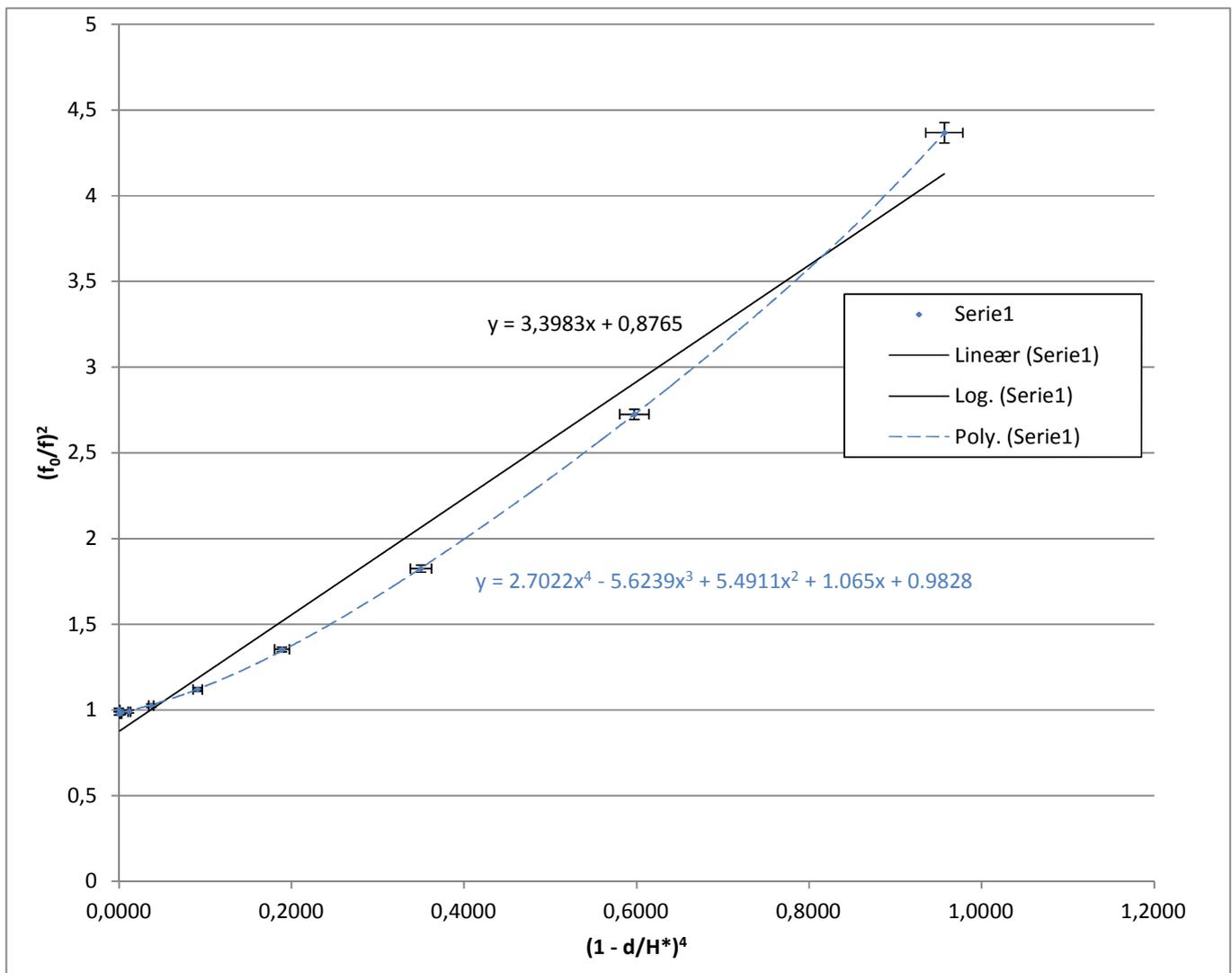
Uncertainty of  $\left( 1 - \frac{d}{H^*} \right)$  is found through:

$$\Delta Q = \frac{d}{H^*} \left( \frac{\Delta d}{d} + \frac{\Delta H^*}{H^*} \right)$$

Then finding the uncertainty of  $\left( 1 - \frac{d}{H^*} \right)^4$  will be:

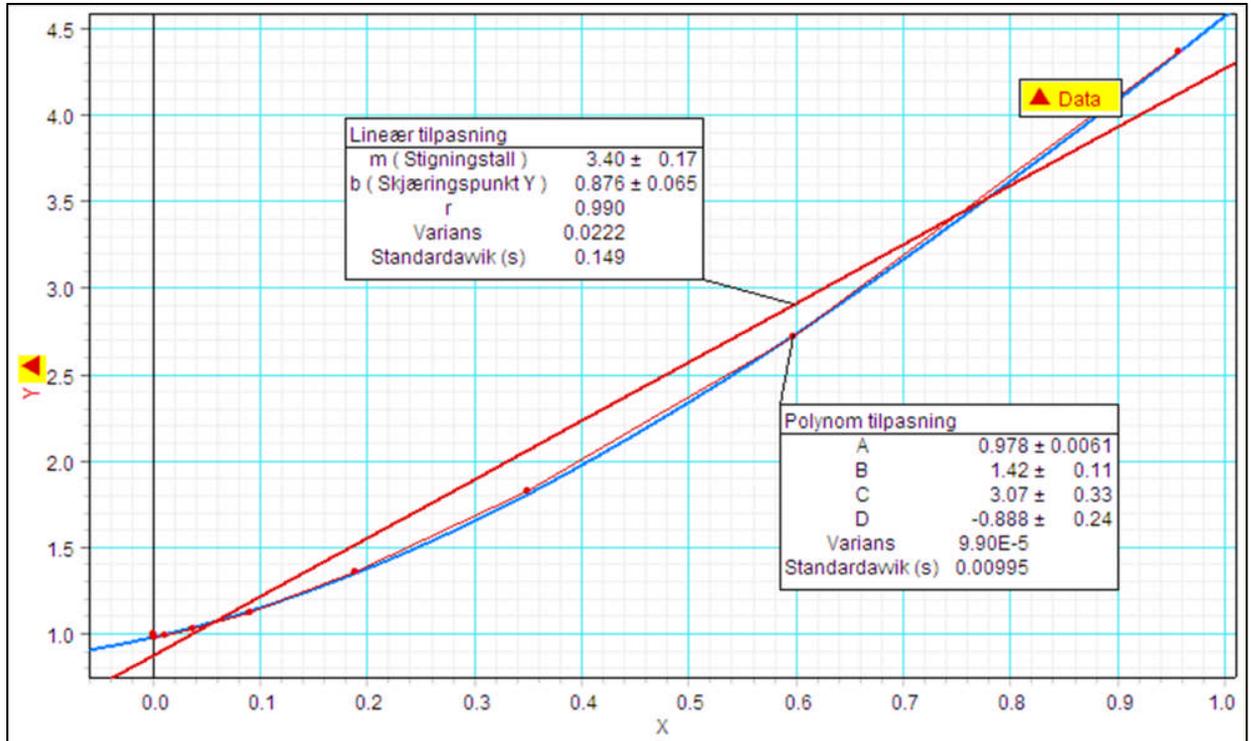
$$\Delta Q = \left( 1 - \frac{d}{H^*} \right) \left( 4 \times \left( \frac{\Delta \left( 1 - \frac{d}{H^*} \right)}{\left( 1 - \frac{d}{H^*} \right)} \right) \right)$$

Then plotting the graph



Graph 2 -  $(f_0/f)^2$  plotted against  $(1 - d/H^*)^4$

Plotting the graph into DataStudio<sup>20</sup>, we can use its regression functions to find the standard deviation for the data points to the trend line:



Graph 3 - Data plotted into DataStudio, everything is in norwegian

The equations of the two regression lines can be found in *Graph 2*.

The standard deviation for the linear best fit is 0.149, while it is 0.00995 for a polynomial regression.

<sup>20</sup> <http://www.pasco.com/datastudio/>

## Analysis

One observation from the results is that the frequencies of the wineglass with water at the bottom decreased very slowly, this is because the bottom part of the wineglass has the stem to reduce the distance the glass can vibrate. The wavelength gets smaller, and according to the equation  $f = \frac{1}{T}$ , the frequency will become large the smaller the wavelength. There is more space for the glass walls to vibrate the further up we go, so the frequency will decrease. We get the “space” from the shape of the glass, the radius of the top is way bigger than at the bottom, and the glass walls are thinner at the top. This will give the glass a more elastically property at the top, and therefore make the walls to vibrate more at the top. This can be proved mathematically through French’s general equation.

We plotted our results in French’s equation and drew the graph of it as we did in *Graph 2*. According to French, an ideal wineglass should have the data points that could have a perfect linear trend line, but the linear trend line we drew did not fit our results perfectly. Our data points showed a slightly curved trend line, as the blue line in *Graph 2* showed, this means that its gradient increases with the x-values. This can be explained when we look at French’s equation<sup>21</sup>

$$\left(\frac{f_0}{f_d}\right)^2 \approx 1 + \frac{\beta\rho_l R}{5\rho_g a} \left(1 - \frac{d}{H^*}\right)^4$$

We only need to look at  $R$  and  $a$  which are the radius of the glass and the thickness of the glass at the water level. The density of the glass and the liquid is constant, since we made sure that the temperature of the water used was approximately the same.  $\beta$  is just a constant he used. In our case, only  $R$  and  $a$  was varying, the further up the water level is the bigger area it will have, thus increasing  $R$  due the shape of the wineglass as seen in *Figure 1*. The thickness of the glass  $a$  will decrease the further up the glass walls, again because of the shape. This will give  $\frac{R}{a}$  an increasing value the more water we have, and thus give the increasing gradient of the trend line, and explains the curved shape of it.

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<sup>21</sup> In *Vino Veritas*, page 692

We have as well found how much the linear line does not fit the general equation of French's, with a standard deviation of 0.149. This is not a big number, but big enough to show that other varying factors are involved in this experiment, like the thickness and radius of the glass at the water level.

## Conclusion

We have now investigated how the frequency varies of a singing wine glass, and how the singing wine glass actually works as well as why the frequency decreases as it does. The results showed that the frequency decreases the more water we add up in the wine glass, however it decreases the most at the upper part of the wineglass, while it nearly does not decrease at the bottom. This is definitely dependent on the shape of the wineglass tested. Because of its shape, the frequency decrease did not fit the general equation of A.P. French. We found out that it was the radius of the glass and the thickness of the glass walls at the different water levels that contributed to a slightly curve when we tried to plot  $\left(\frac{f_0}{f}\right)^2$  against  $\left(1 - \frac{d}{H^*}\right)$ . This explains how and why the frequency decreases the way it does.

## Further investigation

Original intentions were to test out different liquid types with different density, if that will effect this relation or not. Say using yogurt will maybe give different results than using water. The temperature of the liquid may also affect the results due to different density of water at different temperatures, so trying out hot and cold water will also be very interesting. Testing out the frequency change with different types of wine would also be fascinating, like white and red wine, old and new wine. Different liquor and alcohol can also be tried out, since, after all, this is a glass specifically made for drinking alcohol!

## Evaluation:

This experiment has many flaws; how you rub the rim is a great limitation to this investigation. If you rub it too hard, the sound would not be coherent, if you rub it too

soft, then you would not get enough of the stick effect to create a clear sound. Every time I rubbed the glass, there were always some differences, so these uncertainties will definitely affect the frequency outcome. The frequency is found by analysing the sound over a period of time and use the peak of intensity, if the time interval was bigger, the frequency outcomes would be more accurate. However, when measuring the sound, there was only one microphone placed on one side of the wineglass, which produced a beating, so if we have placed several microphones around the wineglass and recorded with all of them, we might eliminate the beating.

Even though the input of the EDIROL was controlled to minimize the interference, there may be interference from my side, rubbing, or random noises, tried to control by analyzing the sound differently. Breathing was sometimes even noticed by the EDIROL.

When the water level was at the top, even analyzing with 4sec time period was too big. There was too much interference of the water that was climbing on to the rim and down the glass walls.

I held the glass stem while rubbing, this might have some impact on the results, and so if the glass fastened on the foot to the table or anything, some small interference could be cancelled. Of course, if a set up that allowed a robot to rub the rim and, but I have done the best out of it already.

## Appendix

### Tables of raw data:

Height (mm)

2. measurement	Peak 1 (Hz)	Peak 2 (Hz)
10	796.1	799.4
20	796.0	798.7
30	791.7	794.6
40	778.9	782.3
50	746.1	749.6
60	675.1	678.8
70	585.4	588.9
80	479.7	482.0
90	377.0	380.3
3. measurement		
10	795.9	799.4
20	795.0	798.5
30	791.4	795.1
40	777.7	781.4
50	744.2	748.0
60	678.4	682.2
70	580.5	584.5
80	473.0	476.2
90	375.8	379.4

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### List of figures:

Figure 2 - A wine glass .....	4
Figure 3 - Superposition.....	5
Figure 4 - How the glass vibrates .....	7
Figure 5 - A vibrating glass with water in action.....	8
Figure 6 - Amplitude - frequency graph .....	8
Figure 7 - Resonance frequencies from 50mm, 3.measurement.....	8
Figure 8 - Beating .....	9
Figure 9 - Measuring instrument, spaghetti .....	13
Figure 11 - The two peaks of 60 mm height, 3. measurement.....	14

Figure 10 - The equipments ..... 14  
 Figure 12 - How the measurements were taken..... 15  
 Figure 13 - How the wave moves ..... 16

List of Graphs:

Graph 1 - Average of both peaks, height as independent variable ..... 18  
 Graph 2 -  $(f_0/f)^2$  plotted against  $(1 - d/H^*)^4$  ..... 20  
 Graph 3 - Data plotted into DataStudio, everything is in norwegian..... 21

List of tables:

Table 1 - Explanation of the signs used in the equation by A.P. French ..... 12  
 Table 2 - Raw data with height as independent variable: ..... 16  
 Table 3 - Processed data with height as independent variable..... 17  
 Table 4 - Processed data 2 ..... 19